JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 15, No. 2, March-April 1992

Reduced-Bandwidth Compensator Design via Control and Observation Normalization

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This paper considers the problem of reduced-bandwidth compensator design for optimal control and estimation systems and presents a new method for linear quadratic Gaussian compensator design using structured control and observation constraints. These constraints tend to normalize the control and observation effort, providing indirect control over the compensator poles, bandwidth, and closed-loop singular values. An example derived from flexible spacecraft control illustrates the procedure.

I. Introduction

COMPENSATOR constraint procedures that enhance robustness are of much interest in the controls community.[1] If a design is not robust (i.e., cannot tolerate system uncertainty), small structural modeling errors can result in large deteriorations in performance and stability margins. Various procedures for achieving and measuring overall system robustness are currently under development such as the H-∞ and singular value techniques.[2] The proposed design method improves robustness indirectly by constraining gains and reducing bandwidth.

With regard to the linear quadratic Gaussian (LQG) methods, i.e., linear optimal control systems employing an estimator, loop transfer recovery (LTR) techniques have been developed to recapture system robustness.[3] The fact that estimator-based compensation destroys the robustness properties of the optimal regulator was demonstrated in Ref. 4, and the LQG/LTR technique was developed to recapture the stability margins of the original state feedback system by rendering the estimator-based portion of the control compensator transparent from the robustness perspective.[5] It is noted that the LTR procedures and LQG compensators, in general, can result in unstable high bandwidth compensation.[1] Such compensation may cause noise problems and poor parametric sensitivity. Compensator bandwidth reduction is essential for rejecting high frequency disturbances that may infringe on the control system. The improved system robustness that accompanies bandwidth reduction is also critical to control system performance and stability.

The control normalization procedure presented here focuses on the LQG compensator dynamic properties, i.e., poles and bandwidth, and attempts to achieve system performance with a low bandwidth stable compensator. Our design philosophy is analogous to that of classical control, i.e., given two closed-loop systems of similar performance, the system with lower gains and lower bandwidth tends to have superior stability margins. The compensator design philosophy is illustrated in Fig. 1 where the outer boundary represents the pole constellation of the unnormalized compensator.

As shown, the unconstrained system tends to have a high bandwidth and in many instances can be unstable. The normalization procedure provides a method to reduce the bandwidth and stabilize the compensator. This procedure involves the incorporation of state control and observation constraints directly in the performance index and is a direct extension of the model error sensitivity suppression (MESS) methods.[6,7] The organization of the paper is as follows: separation principle compensator design is reviewed and followed by a discussion of the normalization procedure and its structured performance index constraints. An example illustrates the procedure's effects on compensator bandwidth and closed-loop singular values.

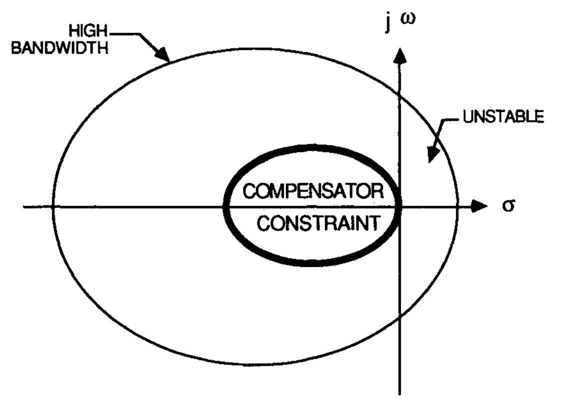

Fig. 1 Philosophy—compensator constraint.

II. Separation Principle Design

Standard textbook optimal control and estimation theory involves the two-step procedure shown in Table 1: one first designs a controller with eigenvalues $[A-BK]$; next one designs an estimator (in a dual control problem) with eigenvalues $[A-GC]$. The separation principle is then invoked to guarantee that the closed-loop system poles are the poles of the controller and the poles of the estimator, i.e., the separation

Table 1 Textbook optimal control and estimation theory

Control		Estimation	
Performance index		Performance index	
$J_1 = \int_0^\infty (x^T Q x + u^T R u)\, \mathrm{d}t$	(1)	$J_2 = \int_0^\infty (z^T \hat{Q} z + \hat{u}^T \hat{R} \hat{u})\, \mathrm{d}t$	(1)
Plant dynamics		Estimator dynamics (dual system)	
$\dot{x} = Ax + Bu$	(2)	$\dot{z} = A^T z + C^T \hat{u}$	(2)
State feedback		State feedback	
$u = -Kx$	(3)	$\hat{u} = -G^T z$	(3)
Controller dynamics		Estimator dynamics	
$A - BK$	(4)	$A - GC$	(4)

Presented as Paper 89-3431 at the AIAA Guidance, Navigation, and Control Conference, Boston, MA, Aug. 14-16, 1989; received Dec. 7, 1989; revision received July 24, 1990; accepted for publication Oct. 24, 1990.

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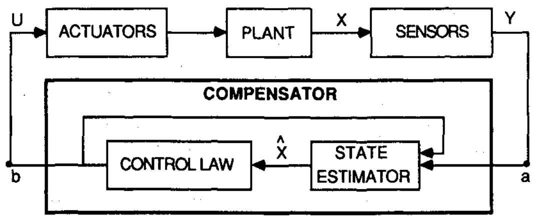


Fig. 2 Combined control and estimation block diagram.

principle allows independent design of control and estimation laws such that no root migration occurs in the controller and estimator subsystems. The result follows from the structure of the closed-loop system matrix when the system dynamics are expressed in terms of the plant states x and the error states e [Eq. (1)]. (The error vector e equals the difference between the state vector x and the estimated state vector $\hat{x}$.) Eigenvalue separation occurs because of the zero submatrix in the lower left-hand corner of the combined system matrix—it prevents closed-loop eigenvalue migration:

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & -BK \\ 0 & A - GC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \tag{1}$$

Figure 2 illustrates the block diagram of the closed-loop system: the controller and estimator form the feedback path around the plant that includes the actuators and sensors. The compensator is the dynamic system that has the observation vector y as its input and the control vector u as its output. The transfer function matrix between points a and b defines the compensator. We denote the compensator by $H(s)$; i.e., $u(s) = H(s)y(s)$, where s is the complex Laplace transform parameter. It can be shown that

$$H(s) = -K(sI - A + BK + GC)^{-1} G \tag{2}$$

where K is the feedback control gain matrix and G is the corresponding estimation gain matrix. The compensator is essentially the state estimator with the feedback loop for the control vector implemented internally. Thus, the LQG design process involves designing a controller with poles given by $[A - BK]$ and an estimator with poles given by $[A - GC]$ but implementation of a compensator with poles given by $[A - BK - GC]$.

The two-step controller-estimator design procedure involving the separation principle completely ignores the compensator—it gives no direct control over compensator pole placement. In some cases an unstable compensation loop results. This situation is not desirable since then the plant is essentially being used to stabilize the controller. Systems that employ unstable compensation are known to be sensitive from classical frequency domain design procedures.[8] This compensator effect is completely masked by the design procedure: the closed-loop system eigenvalues of the controller and estimator have the required stability margins, the nominal system behaves quite well, but the compensator may yield a highly sensitive closed-loop system.

So although the separation principle is one of the strengths of the optimal design procedure in that the poles of the controller and estimator do not migrate, textbook optimal control theory cannot routinely be applied and robust systems automatically generated.

III. Control and Observation Normalization

The control and observation normalization (CON) technique was developed to obtain indirect control over compensator pole placement that standard optimal control and estimation theory does not provide. Application of the CON technique normalizes the control and observation effort through the performance index. Control and observation efforts tend to be uniformly distributed among the appropriate vectors of the actuator and sensor matrices. This is accomplished by penalizing vectors with relatively large magnitudes so that they receive no more control or observation authority than vectors with small magnitudes. The basic principle of the CON technique is the constraint of the internal system driver through the performance index. The performance indices that accomplish this constraint for the controller and the estimator (observer) are shown below as J_1 and J_2, respectively:

$$J_1 = \int_0^\infty [x^T Q x + u^T R u + (Bu)^T R_1 (Bu)] \, dt$$

$$J_2 = \int_0^\infty [z^T \hat{Q} z + \hat{u}^T \hat{R} \hat{u} + (C^T \hat{u})^T \hat{R}_2 (C^T \hat{u})] \, dt \tag{3}$$

The standard LQG performance index is augmented by a control-weighted constraint on the internal driver Bu for the controller or an observation-weighted constraint $C^T\hat{u}$ for the dual estimator.

Collecting terms in u or $\hat{u}$ yields

$$J_1 = \int_0^\infty [x^T Q x + u^T (R + B^T R_1 B) \, u] \, dt$$

$$J_2 = \int_0^\infty [z^T \hat{Q} z + \hat{u}^T (\hat{R} + C \hat{R}_2 C^T) \hat{u})] \, dt \tag{4}$$

where Q, R, and R_1 satisfy well-known existence conditions for optimal solutions.[9] The replacement of R by $[R + B^T R_1 B]$ redefines the control weighting matrix. As R and R_1 are assumed to be positive definite, and $[B^T R_1 B]$ is at least positive semidefinite, the existence conditions are valid.[7] Similar remarks hold for the dual estimator, where the replacement of $\hat{R}$ by $[\hat{R} + C\hat{R}_2 C^T]$ redefines the observation weighting matrix. As shown in the following example, the estimator typically requires more conditioning than the controller.

IV. Design Example—Control and Observation Normalization

The CON procedure will be demonstrated and evaluated with an example derived from flexible spacecraft control. The spacecraft model used in the example is described in Ref. 10. It contains 12 modes with frequencies ranging from 0.67 to 11.4 Hz. Four collocated actuators and sensors were used for control and sensing. CON's effect on bandwidth is evaluated by comparing controller, estimator, and compensator poles before and after applying CON to the full-order model. A singular value analysis evaluates CON's effect on robustness.

Before proceeding directly to a discussion of the design example results, the α-shift technique of Anderson and Moore[11] that is employed to provide a prescribed stability margin for the controller/estimator is briefly described. Figure 3 outlines the α-shift procedure and illustrates the exponentially weighted performance index. The value of α essentially sets the decay envelope for the system and thus is a measure of control (and observation) effort. A prescribed stability margin means that all poles are guaranteed to be to the left of a prescribed vertical line (designated the α line) in the complex plane.

For the design example the α parameter for the controller, α_c, is assigned a value of 0.450. For the estimator, α_e is 10% faster at a value of 0.495. (The desired "α lines" are two times the α values—0.90 for the controller and 0.99 for the estimator.)

Figure 4a shows the controller/estimator designs without CON. The poles of the controller are rather well behaved and seen to approach the 0.9 α line. The control authority appears to be evenly distributed among the modes. For the estimator

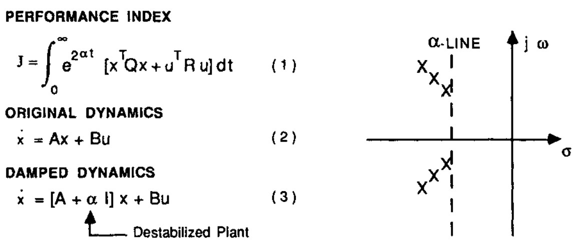
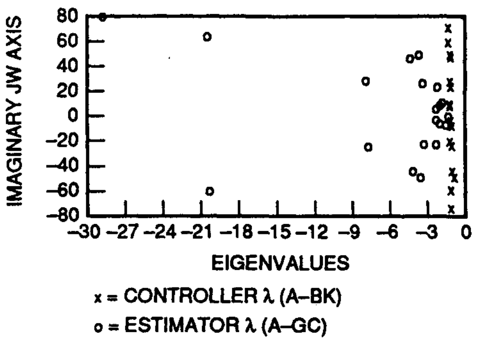
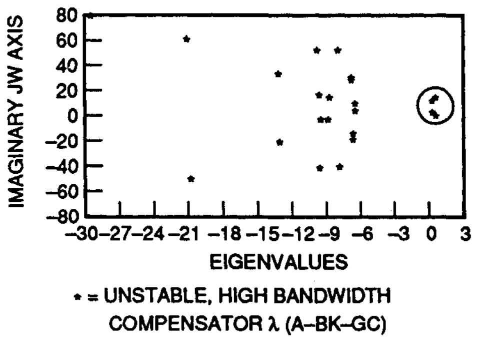


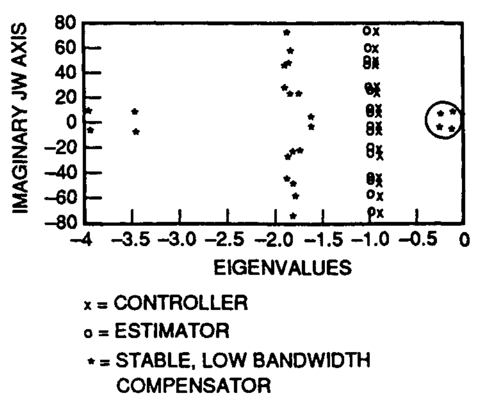
Fig. 3 Prescribed stability margin theory.[11]



a) No normalization



b) No normalization



c) Normalized design

Fig. 4 Normalization design example.

design, though, many poles are to the left of the 0.99 α line. The pole farthest left on the real axis has a real part of -30. Observation authority has been directed at those modes with the highest modal gains. This suggests that most of the compensator improvement will be derived from the observation constraint.

The poles of the controller and estimator without normalization are all stable. However, the compensator that gives those closed-loop poles has two poles that are unstable and a high bandwidth (Fig. 4b).

The CON algorithm is now applied to both the controller and estimator. Figure 4c shows the controller, estimator, and compensator eigenvalues (poles) for the normalized design. The controller poles are forced to more closely approach the 0.9 α line, providing a modest improvement for the well-behaved controller. Estimator poles respond significantly—all are forced to the 0.99 α line. The resulting compensator is stabilized and the real-axis bandwidth drops from -30 to -4. Note that the compensator poles begin to approach a 2(0.45 + 0.495) α line. This can be explained by developing an alternate description for the compensator dynamics. (The derivation is presented in the Appendix.) The compensator dynamics, when written in terms of the Riccati equation parametric matrix, Q instead of R, reveal contributions from three sources: the plant, a scaled identity (diagonal) matrix, and a third matrix designated the residual matrix (the subscripts c and e are associated with the controller and estimator, respectively):

$$A - BR_c^{-1}B^TP_c - P_eBR_e^{-1}B^T = [-A - 2(\alpha_c + \alpha_e)I] - P_c^{-1}[Q_c + A^TP_c] - [Q_e + P_eA^T]\,P_e^{-1} \qquad (5)$$

where Eq. (5) holds for collocated sensors and actuators. Consideration of these matrices indicates a potential source of compensator instability or robustness problems: assuming plant stability, with a small amount of modal damping, the negatively scaled identity matrix increases compensator stability—it shifts the plant eigenvalues left in the complex plane to real parts of $-2(\alpha_c + \alpha_e)$. The remaining residual matrix is potentially responsible for deviations of the eigenvalues from the $-2(\alpha_c + \alpha_e)$ α line. LQG compensators designed with CON are observed to produce smaller residual matrices than their unnormalized counterparts. This accounts for the approach of compensator eigenvalues to values determined by $\lambda[-A - 2(\alpha_c + \alpha_e)I]$ in the example.

A closed-loop singular value analysis of the unnormalized and normalized designs evaluates CON's robustness effect on the system. Figure 5 displays the maximum singular values $\bar{\sigma}$ of the inverse return difference at each modal frequency. (Modal frequency shifts less than 1% were observed between open- and closed-loop systems.) At each modal frequency the maximum singular value for the normalized design is less than that for the unnormalized design—the normalization procedure has produced a 10 to 60 dB decrease in the maximum singular values. Reducing the maximum singular values of the inverse return difference improves disturbance rejection and minimizes sensitivity.

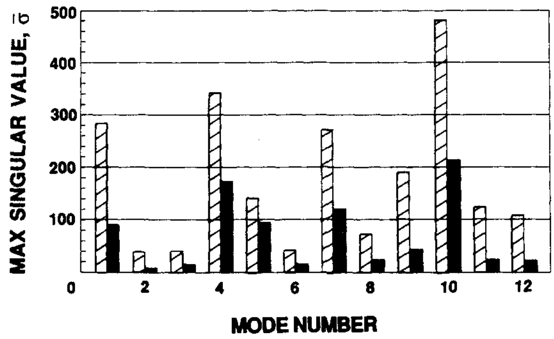


Fig. 5 Maximum singular values $\bar{\sigma}$ of the inverse return difference at each modal frequency.

Analysis of the diagonal elements that form the trace of the estimator GC matrix gives insight into effects of observer normalization. Figure 6 shows the magnitude of these diagonal elements for each mode before and after the CON technique is applied. Before application, the diagonal elements do

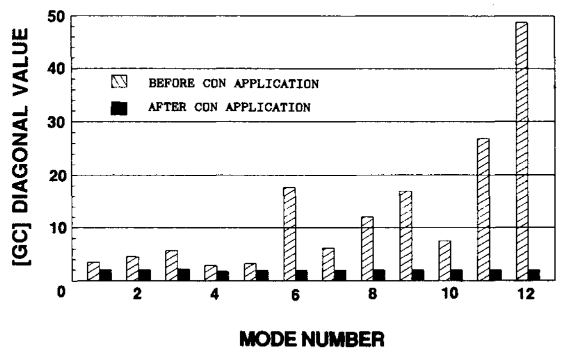


Fig. 6 Normalization effects on diagonal elements of $[GC]$ matrix.

not have a uniform magnitude: values range from 2.8 to 48.7 and roughly correlate with the degree of excess damping beyond the desired α line in the estimator pole constellation of Fig. 4a. After application, the diagonal elements are more uniform with values ranging from 1.8 to 2.2. The trace has been reduced in value from 155.4 to 23.8. The constrained trace approaches a value given by the product of $2\alpha_e$ times the number of modes. Similar changes of reduced magnitude occur for the well-behaved controller.

The CON technique has normalized the control and observation effort through the performance index, forcing uniform distribution of this effort among the appropriate vectors of the optimal gain matrices. The result is a stabilized, reduced-bandwidth compensator, for this example.

V. Conclusions

A new technique for LQG design has been presented that allows indirect control over compensator poles and closed-loop singular values, thereby mitigating some of the robustness deficiencies of the standard LQG separation principle procedure. The technique functions by placing static constraints in the performance index of the optimal controller and dual estimator. By progressively increasing the weighting on these static constraints, under certain conditions, compensator poles are driven toward a constraint region where the compensator's pole constellation tends toward that of the controller plus estimator pole constellation. This improved compensator is generated by constraining control and observation authority via the internal system driver.

Appendix

Derived below is an alternate description for compensator dynamics designed via the alpha-shift technique.[10] When written in terms of the Riccati equation parametric matrix Q instead of R, the compensator dynamics reveal contributions from three sources: the plant, a negatively scaled diagonal matrix, and a residual matrix that is potentially responsible for compensator instability or robustness problems. When the residual matrix is small, the compensator eigenvalues approach the well-behaved eigenvalues of $[-A-2(\alpha_c+\alpha_e)I]$. (The c and e subscripts are associated with the controller and estimator, respectively.) In the following derivation, the standard conditions[9] are assumed to ensure the existence of the unique positive definite solution of the Riccati equation.

Given the controller Riccati equation with plant destabilization parameter α_c

$$[A^T+\alpha_c I]P_c+P_c[A+\alpha_c I]-P_cBR^{-1}B^TP_c+Q_c=0 \quad \text{(A1)}$$

Collecting terms yields

$$A^TP_c+P_cA-P_cBR^{-1}B^TP_c+[Q_c+2\alpha_c IP_c]=0 \quad \text{(A2)}$$

Premultiplying by P_c^{-1} yields

$$P_c^{-1}A^TP_c+A-BR^{-1}B^TP_c+P_c^{-1}[Q_c+2\alpha_c IP_c]=0 \quad \text{(A3)}$$

Collecting terms to isolate the controller closed-loop matrix yields

$$\begin{aligned} A-BR^{-1}B^TP_c &= -P_c^{-1}Q_c-P_c^{-1}A^TP_c-2\alpha_c I \\ &= -P_c^{-1}[Q_c+A^TP_c]-2\alpha_c I \quad \text{(A4)} \end{aligned}$$

Following an analogous procedure for the estimator, and assuming collocated sensors and actuators,

$$\begin{aligned} A-P_eBR^{-1}B^T &= Q_eP_e^{-1}-P_eA^TP_e^{-1}-2\alpha_e I \\ &= -[Q_e+P_eA^T]P_e^{-1}-2\alpha_e I \quad \text{(A5)} \end{aligned}$$

Addition of the controller and estimator equations, Eqs. (A4) and (A5), yields

$$\begin{aligned} &2A-BR^{-1}B^TP_c-P_eBR^{-1}B^T \\ &\quad = -P_c^{-1}[Q_c+A^TP_c]-[Q_e+P_eA^T]P_e^{-1}-2\alpha_c I-2\alpha_c I \quad \text{(A6)} \end{aligned}$$

Subtracting the plant matrix A from both sides of Eq. (A6) and collecting terms on the right-hand side yields the desired expression for the compensator matrix

$$\begin{aligned} &A-BR_c^{-1}B^TP_c-P_eBR_e^{-1}B^T \\ &\quad = [-A-2(\alpha_c+\alpha_e)I]-P_c^{-1}[Q_c+A^TP_c] \\ &\quad -[Q_e+P_eA^T]P_e^{-1} \quad \text{(A7)} \end{aligned}$$

The left side of Eq. (A7) is the usual expression in terms of the Riccati matrices and the parametric design matrices R. The right-hand side is an alternate expression in terms of the weighting matrices Q where the first term in brackets contains the plant plus a diagonal matrix that incorporates the alpha shifts of both the controller and estimator. The matrix containing the remaining right-hand side terms may be considered to be a residual matrix that shifts the compensator poles from the stable eigenvalues by $\lambda[-A-2(\alpha_c+\alpha_e)I]$.

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